Constructing "Reference" Triangle through Unitarity of CKM Matrix

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Abstract

Motivated by the possibility of the low value of $\sin 2\beta$ in the measurements of BABAR and BELLE collaborations, a reference unitarity triangle is constructed using the unitarity of the CKM matrix and the experimental values of the well known CKM elements, without involving any inputs from the processes which might include the new physics effects. The angles of the triangle are evaluated by finding the CP violating phase δ through the Jarlskog's rephasing invariant parameter J. The present data and the unitarity of the CKM matrix gives for δ the range 28° to 152°, which for $\sin 2\beta$ translates to the range 0.21 to 0.88. This range is broadly in agreement with the recent BABAR and BELLE results. However, a value of $\sin 2\beta \leq 0.2$, advocated by Silva and Wolfenstein as a benchmark for new physics, would imply a violation in the three generation unitarity and would hint towards the existence of a fourth generation. Further, the future refinements in the CKM elements will push the lower limit on $\sin 2\beta$ still higher.

The recent measurements of the time dependent CP asymmetry $a_{\psi K_S}$ in $B_d^o(\bar{B}_d^o) \rightarrow \psi K_S$ decay by BABAR and BELLE collaborations, for example,

$$\begin{aligned}
a_{\psi K_S} &= 0.12 \pm 0.37 \pm 0.09 && \text{BABAR [1],} && (1) \\
a_{\psi K_S} &= 0.45^{+0.43}_{-0.44}{}^{+0.07}_{-0.09} && \text{BELLE [2],} && (2)
\end{aligned}$$

look to be smaller compared to the CDF measurements [3], for example,

$$a_{\psi K_S}^{\text{CDF}} = 0.79^{+0.41}_{-0.44} \, , \qquad\qquad (3)$$

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as well as compared to the recent standard analysis of the unitarity triangle [4] with $|\epsilon_K|$, $|\frac{V_{ub}}{V_{cb}}|$, Δm_d and Δm_s as input, given as

$$a_{\psi K_S}^{\text{SM}} = 0.67 \pm 0.17. \tag{4}$$

In the Standard Model, $a_{\psi K_S}$ is related to the angle β of the unitarity triangle as,

$$a_{\psi K_S} = Sin2\beta. \tag{5}$$

Recently, several authors [5] - [7] have explored the implications of the possibility of low value of $\sin 2\beta$ in comparison to the CDF measurements as well as to the global analysis of the unitarity triangle. These analyses lead to the general consensus that the possibility of new physics could be more prominent in the loop dominated processes, in particular the $B_o - \bar{B}_o$ mixing. Further, it is realized that the new physics will not affect the tree level decay processes and the unitarity of the three generation CKM matrix in the SM approaches as well as in its extensions [5]-[12]. In this connection, for better appraisal of new physics, it has been generally recommended to construct a universal or reference unitarity triangle [5],[8]-[12], wherein the inputs are free from the processes which might include the new physics effects, in particular the $B_o - \bar{B}_o$ mixing and $K_o - \bar{K}_o$ mixing parameters. Keeping this in mind several strategies, model dependent [8, 9] as well as model independent [10, 11, 12], have been formulated to construct the triangle, however by and large both approaches rely on the rare decays. The reference triangle to be constructed is defined as,

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0, \tag{6}$$

obtained by employing the orthogonality of the first and third column of the CKM matrix (henceforth referred to as triangle db). In this triangle the elements involving t quark have not been experimentally measured as yet and hence to construct the triangle, the inputs from rare decays involving elements V_{td} and V_{tb} through loops have to be used.

In this context, it is interesting to note that despite several analyses of the CKM phenomenology in the past [4], [13] -[16] yielding valuable information, the implications of three generation unitarity have not been examined in detail in the construction of the reference triangle. A reference triangle constructed purely from the considerations of unitarity as well as using experimentally measured CKM elements will be free from the effects of new physics and hence could serve as a tool for deciphering deviation from the SM in measuring the CP asymmetries.

The purpose of the present communication is to construct the triangle db using unitarity of the three generation CKM matrix by evaluating the Jarlskog's Rephasing Invariant Parameter J and consequently the CP violating

phase δ. In particular, we intend to evaluate angles α, β and γ of the triangle db and study the implications of the low value of $\sin 2\beta$ for unitarity.

To begin with we consider the six non diagonal relations implied by the unitarity of the CKM matrix. One of the relations is mentioned above in equation 6 and the other five are as follows,

$$ds \qquad V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \qquad (7)$$

$$sb \qquad V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \qquad (8)$$

$$ut \qquad V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0, \qquad (9)$$

$$uc \qquad V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0, \qquad (10)$$

$$ct \qquad V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* = 0. \qquad (11)$$

The letters before the different equations denote the respective triangles.

As mentioned above, in the triangle db the elements V_{td} and V_{tb} are not experimentally measured, therefore to obtain these elements without involving inputs from $K_o - \bar{K}_o$ and $B_o - \bar{B}_o$ mixing and rare decays one needs to make use of the PDG [17] representation of the CKM matrix given below,

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \qquad (12)$$

with $c_{ij} = cos\theta_{ij}$ and $s_{ij} = sin\theta_{ij}$ for $i, j = 1, 2, 3$. Since one can obtain s_{12}, s_{23} and s_{13} from the experimentally well known elements $|V_{us}|$, $|V_{cb}|$ and $|\frac{V_{ub}}{V_{cb}}|$ given in Table 1, the CP violating phase δ remains the only unknown parameter in determining the triangle db, which is related to the Jarlskog's rephasing invariant parameter J as,

$$J = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2 sin\delta. \qquad (13)$$

An evaluation of J would allow us to find δ and consequently the angles α, β and γ of the triangle db. To evaluate J, we make use of the fact that the areas of all the six triangles (equations 6-11) are equal and that the area of any of the unitarity triangle is related to Jarlskog's Rephasing Invariant Parameter J as,

$$J = 2 \times \text{Area of any of the Unitarity Triangle.} \qquad (14)$$

This, therefore affords an opportunity to evaluate J through one of the unitarity triangle whose sides are experimentally well known, for example, triangle uc. The triangle uc though is quite well known, but it is highly squashed, therefore one needs to be careful while evaluating J through this triangle. The sides of the triangle represented by $|V_{ud}^*V_{cd}|$ $(= a)$ and $|V_{us}^*V_{cs}|$ $(= b)$ are of

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comparable lengths while the third side $|V_{ub}^* V_{cb}|$ ($= c$) is several orders of magnitude smaller compared to a and b. This creates complications for evaluating the area of the triangle without violating the existence of CP violation. These complications can be avoided without violating the unitarity by incorporating the constraints $|a| + |c| > |b|$ and $|b| + |c| > |a|$ [18]. Using these constraints and the experimental data given in the table 1, a histogram can be generated, shown in figure 1, to which a gaussian is fitted yielding the result,

$$|J| = (2.59 \pm 0.79) \times 10^{-5}. \tag{15}$$

This value of $|J|$ can now be used to calculate δ using the equation 13, which can be re-written as,

$$J = J' sin\delta, \tag{16}$$

where,

$$J' = s_{12} s_{23} s_{13} c_{12} c_{23} c_{13}^2. \tag{17}$$

Calculating s_{12}, s_{23} and s_{13} from the experimental values of $|V_{us}|$, $|\frac{V_{ub}}{V_{cb}}|$, and $|V_{cb}|$ given in table 1 and following the procedure outlined above for evaluating $|J|$, J' comes out to be,

$$J' = (3.23 \pm 0.63) \times 10^{-5}. \tag{18}$$

Since $J' sin\delta$ should reproduce $|J|$ calculated through the unitarity triangle uc, therefore comparing equations 15 and 18, one can easily find out the widest limits on δ, for example,

$$\delta = 28^o \text{ to } 152^o. \tag{19}$$

This value of δ apparently looks to be the consequence only of the unitarity relationship given by equation 10. However on further investigation, as shown by Branco and Lavoura [18], one finds that this δ range is consequence of all the non trivial unitarity constraints. In this sense the above range could be attributed to as a consequence of unitarity of the CKM matrix. It needs to be noted that with the above range of δ and the experimental values of $|V_{us}|$, $|V_{cb}|$ and $|\frac{V_{ub}}{V_{cb}}|$ given in Table 1, the CKM matrix thus evaluated is in excellent agreement with PDG CKM matrix [17].

Alternatively, using equation 16, one can plot a histogram for δ as well, to which fitting a Gaussian yields,

$$\begin{aligned} \delta \ &= \ 50^o \pm 20^o \text{ (I quadrant)}, \\ &\quad 130^o \pm 20^o \text{ (II quadrant)}. \end{aligned} \tag{20}$$

This gives us relatively stronger bounds on δ. However, to be conservative, we have used the range of δ as given by equation 19 for the subsequent calculations.

After having obtained a range for δ, the triangle db can be constructed, however without involving inputs from the phenomena which may have influence from the new physics as well as without the inputs from the rare decays. The angles α, β and γ of the triangle can be expressed in terms of the CKM elements as,

$$\alpha = arg\left(\frac{-V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \tag{21}$$

$$\beta = arg\left(\frac{-V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \tag{22}$$

$$\gamma = arg\left(\frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right), \tag{23}$$

where CKM elements are as given by the PDG representation in the equation 12. In the Table 1 we have listed the experimental values of the CKM elements as given by PDG [17] as well as their future values. Making use of the PDG representation of CKM matrix given in equation 12, experimental values of $|V_{us}|$, $|V_{cb}|$ and $|\frac{V_{ub}}{V_{cb}}|$ from table 1 and the range of δ given by equation 19, one can easily find out the corresponding ranges for the three angles. In the Table 2, we have listed the corresponding results for J, δ, α, β and γ. The ranges for α, β and γ are as follows,

$$\alpha \simeq 19^o \text{ to } 142^o, \tag{24}$$

$$\beta \simeq 6^o \text{ to } 31^o, \tag{25}$$

$$\gamma \simeq 28^o \text{ to } 152^o. \tag{26}$$

While evaluating the three angles, we have taken care that the triangle is closed. The range of $sin2\beta$ corresponding to equation 25 is given as,

$$sin2\beta = 0.21 \text{ to } 0.88. \tag{27}$$

It needs to be emphasized that this range for $sin2\beta$ is obtained by making use of unitarity and the well known CKM elements listed in Table 1. The above range has considerable overlap with the BABAR and BELLE results, however if $sin2\beta$ is found to be ≤ 0.2, a benchmark for new physics as advocated by Silva and Wolfenstein [6], then one may conclude that even the three generation unitarity may not be valid and one may have to go to four generations to explain the low values of $sin2\beta$. In such a scenario, the widely advocated assumption [5] -[12] that the non SM physics resides in loop dominated processes only may not be valid.

A few comments are in order. It is interesting to examine the consequences of the future refinements in the CKM elements. While listing the future values

of the elements we have considered only those elements where the present error is more than 15%, for example $|\frac{V_{ub}}{V_{cb}}|$ and $|V_{cs}|$. The future values of these elements are listed in column III of Table 1. One finds from the Table 2 that the refinements in $|\frac{V_{ub}}{V_{cb}}|$ and $|V_{cs}|$ would improve the lower bound on $\sin2\beta$ from 0.21 to 0.31. This would give a clear signal for physics beyond the SM in case $\sin2\beta$ is measured to be ≤ 0.2. To emphasize this conclusion, we have also considered all the future inputs at their 90% CL and this gives the lower limit of $\sin2\beta$=0.18.

It may be of interest to mention that a recent investigations involving texture 4 zeros quark mass matrices and unitarity [19], yield the following range for $\sin2\beta$,

$$Sin2\beta = 0.27 \text{ to } 0.60, \tag{28}$$

which looks to be compatible with the present unitarity based calculations. A value of $\sin2\beta \leq 0.2$ therefore, will have far reaching consequences for unitarity as well as for texture specific mass matrices [20].

It is interesting to compare our results (equation 27) with those of Buras (equation 4), obtained from the measurements of $|\epsilon_K|$, $|\frac{V_{ub}}{V_{cb}}|$, Δm_d and Δm_s, which look to be much narrower compared to ours. This is easy to understand when one considers the definition of β given in equation 22, wherein the magnitude and phase of V_{td} play an important role. For example, the range of δ given by equation 19 yields the V_{td} range as 0.0045 to 0.0135, whereas the range corresponding to Buras's analysis is 0.0067 to 0.0093, which is narrower primarily due to restrictions imposed by $|\epsilon_K|$, Δm_d and Δm_s.

To conclude, we have constructed a reference unitarity triangle by making use of the three generation unitarity of the CKM matrix and the experimental values of the well known CKM elements, without involving any inputs from the processes which might include the new physics effects, in particular the $B_o - \bar{B}_o$ mixing and $K_o - \bar{K}_o$ mixing parameters as well as the rare decays. The angles of the triangle have been evaluated by finding the CP violating phase δ through the Jarlskog's rephasing invariant parameter J. The range of δ comes out to be 28° to 152° and the corresponding range for $\sin2\beta$ is 0.21 to 0.88. This range is broadly in agreement with the recent BABAR and BELLE results and also has considerable overlap with the range found from the texture 4 zeros quark mass matrices and the unitarity of the CKM matrix. However, a value of $\sin2\beta \leq 0.2$ advocated by Silva and Wolfenstein as a benchmark for new physics would imply a violation in the three generation unitarity and would hint towards the existence of a fourth generation. Further, the future refinements in the CKM elements will push the lower limit on $\sin2\beta$ still higher, for example from 0.21 to 0.31, thus giving a clear signal for physics beyond the SM in case $\sin2\beta$ is measured to be ≤ 0.2. This remains valid even when the future values are considered at their 90% CL.

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Parameter	PDG values [17]	Future values
$|V_{ud}|$	0.9735 ± 0.0008	0.9735 ± 0.0008
$|V_{us}|$	0.2196 ± 0.0023	0.2196 ± 0.0023
$|V_{cd}|$	0.224 ± 0.016	0.224 ± 0.016
$|V_{cs}|$	1.04 ± 0.16	1.04 ± 0.08
$|V_{cb}|$	0.0402 ± 0.0019	0.0402 ± 0.0019
$|\frac{V_{ub}}{V_{cb}}|$	0.090 ± 0.025	0.090 ± 0.010

Table 1: Values of the CKM parameters used throughout the paper.

	With PDG values	With future values	With future values at their 90% CL
J	$(2.59 \pm 0.79) \times 10^{-5}$	$(2.79 \pm 0.49) \times 10^{-5}$	$(2.61 \pm 0.78) \times 10^{-5}$
δ	28° to 152°	42° to 138°	30° to 150°
α	19° to 141°	28° to 124°	19° to 143°
β	6° to 31°	9° to 31°	5° to 36°
γ	28° to 152°	42° to 138°	30° to 150°

Table 2: J, δ and corresponding α, β and γ with PDG and the future values of input parameters listed in table 1
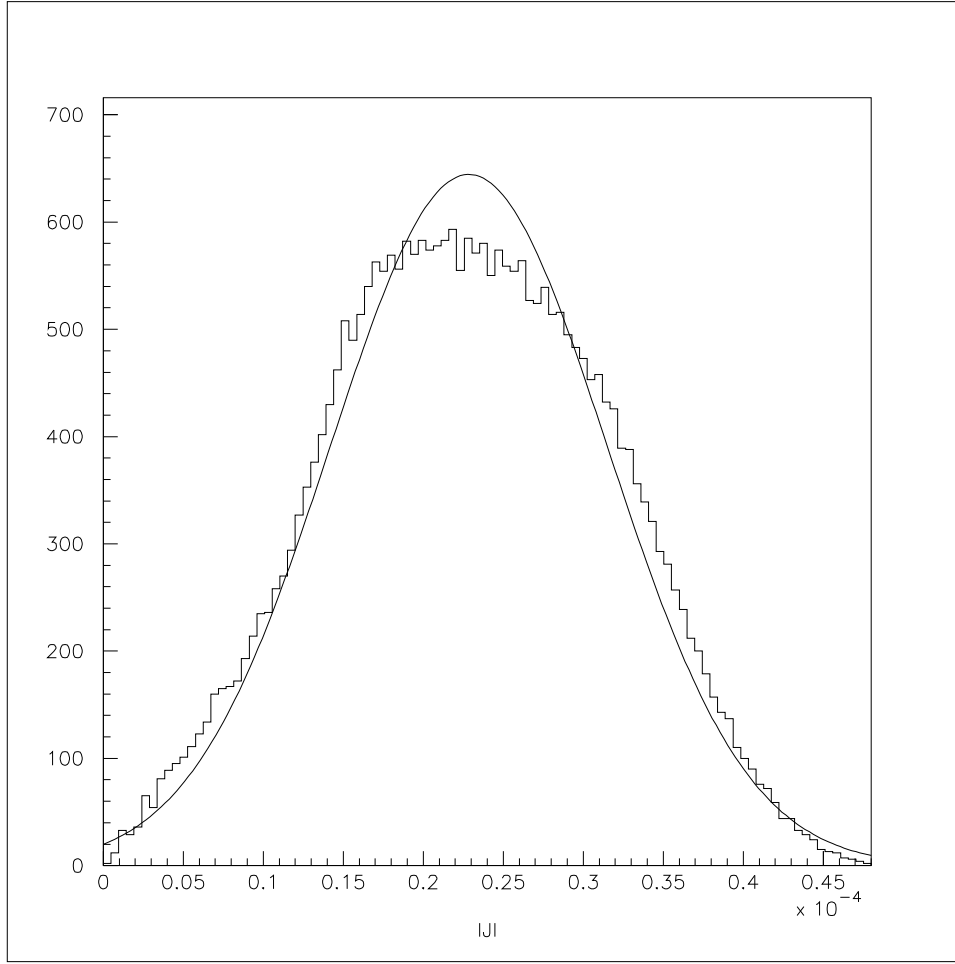
Figure 1: Gaussian fitted to the histogram of $|J|$ generated by considering the triangle uc with the input constraints $|a| + |c| > |b|$ and $|b| + |c| > |a|$, where $a = |V_{ud}^* V_{cd}|$, $b = |V_{us}^* V_{cs}|$ and $c = |V_{ub}^* V_{cb}|$.